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# LEPTONIC QED CORRECTIONS TO THE PROCESS $ep \longrightarrow eX$ IN JAQUET-BLONDEL VARIABLES

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## ABSTRACT

For the study of deep inelastic scattering at HERA, the use of Jaquet-Blondel variables is advantageous in several respects. We calculate the complete leptonic  $\mathcal{O}(\alpha)$  QED corrections for the reaction  $ep \rightarrow eX$  in these variables. All but one phase space integrations are performed analytically. After exponentiation of soft photon corrections, an accuracy of at least 1% is matched. Numerical results are presented and compared to estimates based on the leading logarithmic approximation.

Equation (17) is corrected in accordance with equation (7.44) of DESY 94–115. The misprint did not influence the numerics.

# 1 Introduction

At HERA, the deep-inelastic scattering of electrons off protons,

$$e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2), \quad (1)$$

allows the study both of the basic interactions of electrons and protons and of the proton structure in a completely new kinematic region [1]. Reaction (1) has to be analyzed together with the corresponding photonic corrections from the radiative process,

$$e(k_1) + p(p_1) \rightarrow e(k_2) + X(p_2) + n\gamma(k). \quad (2)$$

Bremsstrahlung may substantially contribute to the observed net cross section. Its amount crucially depends both on the experimental cuts applied to the photonic phase space and on the kinematic variables and their allowed regions of variation. During a longer period, the use of leptonic variables dominated in the literature. At HERA, these have certain disadvantages: QED corrections may become extremely large, and in certain kinematic regions it is impossible to separate energetically the final electron from the bremsstrahlung photon. More advantageous are the hadronic variables on which the structure functions depend. Unfortunately, their experimental determination is also nontrivial or even impossible. A natural compromise has been proposed with the *Jaquet–Blondel* (JB) variables [3]. During the 1991 HERA Workshop [2], features of the various sets of variables have been discussed in detail. Here we will concentrate on the case of JB variables. These variables are not influenced neither by the leptonic nor the longitudinal hadronic degrees of freedom:

$$Q_{\text{JB}}^2 = \frac{(\sum_h \vec{p}_{h\perp})^2}{1 - y_{\text{JB}}} = \frac{(\vec{p}_{2\perp})^2}{1 - y_{\text{JB}}}, \quad y_{\text{JB}} = y_h = \frac{-2p_1 Q_h}{S}, \quad x_{\text{JB}} = \frac{Q_{\text{JB}}^2}{y_{\text{JB}} S}, \quad (3)$$

with  $S = -(k_1 + p_1)^2$  and  $Q_h = p_2 - p_1$ . Using the JB variables, one has the opportunity to search for a phase space parametrization with an early integration over the leptonic degrees of freedom. In another approach, we did not use this opportunity since we tried there to handle leptonic, mixed, and hadronic variables in a common formalism as much as possible [4, 5]. For details of the derivation of the formulae which will be presented here, we refer to that article.

In this letter, we take advantage of the above-mentioned properties of the JB variables. We have performed all but one phase space integrations for the double-differential cross section. We explicitly sketch the rather compact complete  $\mathcal{O}(\alpha)$  *leptonic* corrections from reaction (2) to the double-differential cross section of reaction (1), including soft photon exponentiation. The leptonic corrections arise from the radiation of photons from the leptonic part of the Feynman diagrams. They are much larger than the other ones – quark- and lepton-quark interference bremsstrahlung. Further, leptonic bremsstrahlung may be treated *model-independently*, relying on some set of structure functions which need not necessarily be related to the quark-parton model.

In Section 2, the kinematics is explained and the basic formulae are collected. Section 3 contains numerical results and their discussion.

## 2 Kinematics and phase space integrations

### 2.1 The Born cross section

The cross section of (1) is

$$\frac{d^2\sigma_B}{dQ^2 dy} = \frac{2\pi\alpha^2}{SQ^4} \sum_{i=1}^3 A_i(x, Q^2) \mathcal{S}_i^B(Q^2, y), \quad (4)$$

where, of course, for the lowest-order cross section the different sets of kinematic variables agree; e.g.  $Q_{JB}^2 = Q_h^2$ ,  $y_{JB} = y_h$ ,  $x_{JB} = x_h$ . Further,

$$\begin{aligned} \mathcal{S}_1^B &= Q^2, \\ \mathcal{S}_2^B &= 2(1-y)S^2, \\ \mathcal{S}_3^B &= 2(2-y)Q^2S. \end{aligned} \quad (5)$$

The  $A_i$  are proportional to the structure functions  $\mathcal{F}_{1,2,3}^{\text{NC}}(x_h, Q_h^2)$  describing the electroweak interactions of leptons and nucleons; their arguments have to be defined by the hadronic kinematics:

$$\begin{aligned} A_1 &\equiv (2MW_1) = \mathcal{F}_1^{\text{NC}}(x_h, Q_h^2), \\ A_2 &\equiv \frac{1}{y_h S}(\nu W_2) = \frac{1}{y_h S} \mathcal{F}_2^{\text{NC}}(x_h, Q_h^2), \\ A_3 &\equiv \frac{1}{2y_h S}(\nu W_3) = \frac{1}{2y_h S} \mathcal{F}_3^{\text{NC}}(x_h, Q_h^2). \end{aligned} \quad (6)$$

The structure functions are defined as usually:

$$\begin{aligned} \mathcal{F}_{1,2}^{\text{NC}}(x, Q^2) &= F_{1,2}(x, Q^2) + 2|Q_l|v_l\chi G_{1,2}(x, Q^2) + \chi^2(v_l^2 + a_l^2)H_{1,2}(x, Q^2), \\ \mathcal{F}_3^{\text{NC}}(x, Q^2) &= -2\chi Q_l a_l G_3(x, Q^2) - 2\chi^2 Q_l v_l a_l H_3(x, Q^2), \end{aligned} \quad (7)$$

where  $Q_l$ ,  $v_l$  and  $a_l$  are the corresponding charge ( $Q_{l\pm} = \pm 1$ ), vector and axial-vector couplings of the lepton with the  $Z$  boson:  $v_l = 1 - 4|Q_l|\sin^2\theta_w$ ,  $a_l = 1$ , and  $\theta_w$  is the weak mixing angle. Further,

$$\chi \equiv \chi(Q^2) = \frac{G_\mu}{\sqrt{2}} \frac{M_Z^2}{8\pi\alpha} \frac{Q^2}{Q^2 + M_Z^2}. \quad (8)$$

The  $F, G, H$  are structure functions parametrizing the general hadronic tensor for the  $|\gamma|^2, \gamma Z$ , and  $|Z|^2$  contributions to the Born cross-section, respectively.

For the radiative process, the sum at the right-hand side in (4) will become the integrand of a three-dimensional phase space integral. Further, the kinematic weights  $\mathcal{S}_i$  which accompany the structure functions become complicated expressions of the invariants. The specific problem which has to be solved for each set of variables separately is the subsequent analytic integration. In the next chapter, we comment on that in terms of Jaquet–Blondel (JB) variables and perform two integrations explicitly. Then, we are left with a one-dimensional integral which has to be performed numerically.

## 2.2 The phase space integration

With JB variables, the following parametrization of the phase space may be derived <sup>1</sup> [5]:

$$\Gamma = \frac{\pi^2}{4} \int dQ_{\text{JB}}^2 dy_{\text{JB}} \int_{m^2}^{\tau_{\text{max}}} d\tau \frac{1}{4\pi} \frac{\tau - m^2}{\tau} \int_{-1}^1 d \cos \vartheta_R \int_0^{2\pi} d\varphi_R \int d\Gamma_h. \quad (9)$$

Here, the

$$d\Gamma_h = \prod_i \frac{d\vec{p}_i}{2p_i^0} \delta^{(4)}(p_2 - \sum_i p_i) \quad (10)$$

is the phase space element of the final hadron system which will be completely absorbed into the definition of the hadronic structure functions. Further, we use the invariant mass  $\tau = -(k_2 + k)^2$  of the  $(e, \gamma)$  compound and the photonic angles  $\vartheta_R$ ,  $\varphi_R$  in the rest system of this compound, defined by  $\vec{k}_2 + \vec{k} = 0$ . Further,  $m$  is the electron mass, and the upper limit of the  $\tau$  variation is

$$\tau_{\text{max}} = (1 - x_{\text{JB}})(1 - y_{\text{JB}})S. \quad (11)$$

In these variables, the double-differential cross section becomes:

$$\frac{d^2\sigma_R}{dQ_{\text{JB}}^2 dy_{\text{JB}}} = \frac{2\alpha^3}{S} \int d\tau \sum_{i=1}^3 \frac{1}{Q_h^4} A_i(x_h, Q_h^2) \mathcal{S}_i(Q_{\text{JB}}^2, y_{\text{JB}}, \tau), \quad (12)$$

$$\mathcal{S}_i(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) = \frac{1}{4\pi} \frac{\tau - m^2}{\tau} \int d \cos \vartheta_R d\varphi_R \mathcal{S}_i(Q_{\text{JB}}^2, y_{\text{JB}}, \tau, \cos \vartheta_R, \varphi_R). \quad (13)$$

The functions  $\mathcal{S}_i(Q_{\text{JB}}^2, y_{\text{JB}}, \tau, \cos \vartheta_R, \varphi_R)$  are calculated from the squared sum of the Feynman diagrams. Their explicit expressions may be found elsewhere [6, 5]. In the derivation of (12) we took advantage of the fact that the hadronic variables, and thus the hadronic structure functions, are independent of the photonic angles. The hadronic transferred momentum squared  $Q_h^2$  and  $Q_{\text{JB}}^2$  are related by the following relation containing the additional variable  $\tau$ :

$$Q_h^2 \equiv (p_2 - p_1)^2 = Q_{\text{JB}}^2 + \frac{y_{\text{JB}}}{1 - y_{\text{JB}}}(\tau - m^2). \quad (14)$$

For Born kinematics,  $\tau$  approaches its lower limit and  $Q_h^2$  and  $Q_{\text{JB}}^2$  agree. The integral over  $\tau$  in (12) has to be performed numerically as long as one cannot neglect the difference between  $Q_{\text{JB}}^2$  and  $Q_h^2$  – the structure functions are depending on  $Q_h^2$  and thus also on  $\tau$ . At the other hand, the integral over the photonic angles may be performed analytically. For details we refer to [7, 5] where the integration techniques are explained and most of the integrals used may be found. Again, we quote here only the final result which has been obtained by means of the program for analytical calculations **SCHOONSCHIP** [8]:

$$\mathcal{S}_1(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) = Q_{\text{JB}}^2 \left[ \frac{1}{z_2} (L_\tau - 2) + \frac{z_2}{4\tau^2} \right] + \frac{1 - 8y_{\text{JB}}}{4(1 - y_{\text{JB}})} + L_\tau \left( \frac{z_2}{2Q_\tau^2} + \frac{y_{\text{JB}}}{1 - y_{\text{JB}}} \right), \quad (15)$$

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<sup>1</sup>Whenever necessary, we retain the electron mass  $m$  although the ultra-relativistic limit is assumed.

$$\begin{aligned} \mathcal{S}_2(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) &= S^2 \left\{ 2(1 - y_{\text{JB}}) \left[ \frac{1}{z_2} (L_\tau - 2) + \frac{z_2}{4\tau^2} \right] - \frac{1}{Q_\tau^2} \left[ 2(L_\tau - 2) + \frac{1}{2}(1 - y_{\text{JB}}^2) \right] \right. \\ &\quad \left. + \frac{Q_{\text{JB}}^2}{Q_\tau^4} (1 - y_{\text{JB}}) [1 - (1 + y_{\text{JB}})(L_\tau - 3)] - \frac{Q_{\text{JB}}^4}{Q_\tau^6} (1 - y_{\text{JB}})^2 (L_\tau - 3) \right\}, \quad (16) \end{aligned}$$

$$\begin{aligned} \mathcal{S}_3(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) &= S \left\{ 2Q_{\text{JB}}^2 (2 - y_{\text{JB}}) \left[ \frac{1}{z_2} (L_\tau - 2) + \frac{z_2}{4\tau^2} \right] + \frac{y_{\text{JB}}(1 + y_{\text{JB}}^2)}{1 - y_{\text{JB}}} L_\tau + 5 \right. \\ &\quad \left. - \frac{7y_{\text{JB}}}{2(1 - y_{\text{JB}})} - (1 - y_{\text{JB}})(5 + 2y_{\text{JB}}) \right. \\ &\quad \left. + \frac{Q_{\text{JB}}^2}{Q_\tau^2} (1 - y_{\text{JB}}) \left[ (1 - y_{\text{JB}}) \left( 3 - \frac{2Q_{\text{JB}}^2}{Q_\tau^2} \right) (L_\tau - 2) + 12 - 5L_\tau \right] \right\}, \quad (17) \end{aligned}$$

where

$$z_2 = -2k_2 k = \tau - m^2, \quad (18)$$

$$L_\tau = \ln \frac{Q_\tau^2}{m^2 \tau}, \quad (19)$$

$$Q_\tau^2 = Q_{\text{JB}}^2 + \frac{\tau - m^2}{1 - y_{\text{JB}}}. \quad (20)$$

### 2.3 The infra-red problem and soft photon exponentiation

The integral (12) diverges at  $z_2 = 0$  ( $k = 0$ ). This may be seen from (15–17); the kinematic factors  $\mathcal{S}_i$  contain the Born functions (5) together with the common, divergent factor  $(L_\tau - 2)/z_2$ . The infra-red divergent part of (12) may be isolated:

$$\frac{d^2 \sigma_{\text{R}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} = \left[ \frac{d^2 \sigma_{\text{R}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} - \frac{d^2 \sigma_{\text{R}}^{\text{IR}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} \right] + \frac{d^2 \sigma_{\text{R}}^{\text{IR}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} \equiv \frac{d^2 \sigma_{\text{R}}^{\text{F}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} + \frac{d^2 \sigma_{\text{R}}^{\text{IR}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}}. \quad (21)$$

Here  $d^2 \sigma_{\text{R}}^{\text{F}}/dQ_{\text{JB}}^2 dy_{\text{JB}}$  is finite at  $k \rightarrow 0$ ; it is defined as follows:

$$\begin{aligned} \frac{d^2 \sigma_{\text{R}}^{\text{F}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} &= \frac{2\alpha^3}{S} \int d\tau \sum_{i=1}^3 \left[ \frac{1}{Q_h^4} A_i(x_h, Q_h^2) \mathcal{S}_i(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) \right. \\ &\quad \left. - \frac{1}{Q_{\text{JB}}^4} A_i(x_{\text{JB}}, Q_{\text{JB}}^2) \mathcal{S}_i^{\text{B}}(Q_{\text{JB}}^2, y_{\text{JB}}) F^{\text{IR}}(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) \right], \quad (22) \end{aligned}$$

$$F^{\text{IR}}(Q_{\text{JB}}^2, y_{\text{JB}}, \tau) = \frac{1}{z_2} (L_\tau - 2) - \frac{1}{(1 - y_{\text{JB}}) Q_\tau^2} L_\tau + \frac{1}{\tau}. \quad (23)$$

From the above equations, it is easy to derive an explicit expression for the isolated infra-red divergent part  $d^2 \sigma_{\text{R}}^{\text{IR}}$ . It contains an integral over  $\tau$  which may be performed explicitly using dimensional regularization [9, 5].

Finally, the resulting cross section is

$$\frac{d^2 \sigma}{dQ_{\text{JB}}^2 dy_{\text{JB}}} = \frac{d^2 \sigma_{\text{R}}^{\text{F}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} + \frac{d^2 \sigma_{\text{B}}}{dQ_{\text{JB}}^2 dy_{\text{JB}}} \left[ 1 + \frac{\alpha}{\pi} \delta^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) \right], \quad (24)$$

$$\delta^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) \equiv \delta_{\text{vert}}(Q_{\text{JB}}^2, y_{\text{JB}}) + \delta_{\text{R}}^{\text{IR}}(Q_{\text{JB}}^2, y_{\text{JB}}). \quad (25)$$

As mentioned, from (22) one derives

$$\delta_{\text{R}}^{\text{IR}}(Q_{\text{JB}}^2, y_{\text{JB}}) = \int d\tau F^{\text{IR}}(Q_{\text{JB}}^2, y_{\text{JB}}, \tau), \quad (26)$$

with  $F^{\text{IR}}$  defined in (23). The QED vertex correction  $\delta_{\text{vert}}$  compensates for the infra-red divergence  $\mathcal{P}^{\text{IR}}(\mu)$  of the soft bremsstrahlung contribution,

$$\delta_{\text{vert}}(Q_{\text{JB}}^2, y_{\text{JB}}) = -2\mathcal{P}^{\text{IR}}(\mu) (L_{\text{JB}} - 1) - \frac{1}{2}L_{\text{JB}}^2 + \frac{3}{2}L_{\text{JB}} + \text{Li}_2(1) - 2, \quad (27)$$

where

$$L_{\text{JB}} = \ln \frac{Q_{\text{JB}}^2}{m^2}, \quad (28)$$

and  $\text{Li}_2(1)$  is the Euler dilogarithm. The net photonic correction  $\delta^{\text{VR}}$  is finite:

$$\begin{aligned} \delta^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) &= \delta^{\text{inf}}(Q_{\text{JB}}^2, y_{\text{JB}}) - \ln^2 X_{\text{JB}} + [\ln(1 - y_{\text{JB}}) - 1] \ln X_{\text{JB}} \\ &\quad - \frac{1}{2} \ln^2 \left[ \frac{(1 - x_{\text{JB}})(1 - y_{\text{JB}})}{x_{\text{JB}} y_{\text{JB}}} \right] \\ &\quad + \frac{3}{2} L_{\text{JB}} - \text{Li}_2(X_{\text{JB}}) - 2\text{Li}_2(1 - X_{\text{JB}}) - 1, \end{aligned} \quad (29)$$

where

$$\delta^{\text{inf}}(Q_{\text{JB}}^2, y_{\text{JB}}) = (L_{\text{JB}} - 1) \ln \frac{(1 - x_{\text{JB}})(1 - y_{\text{JB}})}{1 - x_{\text{JB}}(1 - y_{\text{JB}})}, \quad (30)$$

$$X_{\text{JB}} = \frac{1}{x_{\text{JB}} y_{\text{JB}}} [1 - x_{\text{JB}}(1 - y_{\text{JB}})]. \quad (31)$$

Finally, the effect of multiple soft photon emission may be taken into account by soft photon exponentiation:

$$\begin{aligned} \frac{\alpha}{\pi} \delta^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) &\rightarrow \frac{\alpha}{\pi} \delta_{\text{exp}}^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) \\ &= \frac{\alpha}{\pi} [\delta^{\text{VR}}(Q_{\text{JB}}^2, y_{\text{JB}}) - \delta^{\text{inf}}(Q_{\text{JB}}^2, y_{\text{JB}})] + \left\{ \exp \left\{ \frac{\alpha}{\pi} \delta^{\text{inf}}(Q_{\text{JB}}^2, y_{\text{JB}}) \right\} - 1 \right\}. \end{aligned} \quad (32)$$

### 3 Results and discussion

The numerical results are obtained with an extended version of the FORTRAN program TERAD91 [5]. We use exactly the same specifications of kinematics, structure functions, weak Standard Model parameters for the  $Z$  amplitude, etc. as have been used at the 1992 HERA workshop [10]. For this reason, the running of the QED coupling has not been taken into

$x_{\text{JB}}$	$y_{\text{JB}}$	TERAD complete $\mathcal{O}(\alpha)$	HELIOS LLA $\mathcal{O}(\alpha)$
$10^{-3}$	0.01	-0.19	-0.01
	0.10	-1.17	-1.1
	0.50	-4.59	-4.5
	0.90	-12.11	-11.7
	0.99	-24.15	-22.2
$10^{-2}$	0.01	-0.22	-0.2
	0.10	-1.49	-1.4
	0.50	-5.50	-5.3
	0.90	-13.98	-13.5
	0.99	-27.31	-25.2
0.10	0.01	-0.29	-0.3
	0.10	-1.82	-1.8
	0.50	-6.19	-6.0
	0.90	-15.86	-15.2
	0.99	-30.57	-28.4
0.50	0.01	-0.58	-0.5
	0.10	-3.29	-3.1
	0.50	-11.11	-10.7
	0.90	-24.17	-22.8
	0.99	-40.75	-37.2

Table 1: *Leptonic radiative corrections  $\delta$  of  $\mathcal{O}(\alpha)$  in percent; first column this calculation, second column a LLA calculation [11].*

account in the matrix elements. We use first the  $\mathcal{O}(\alpha)$  approximation. The table 1 contains the QED corrections  $\delta$ , which are defined as follows:

$$\delta = \left[ \frac{d^2\sigma/dQ_{\text{JB}}^2 dy_{\text{JB}}}{d^2\sigma_{\text{B}}/dQ_{\text{JB}}^2 dy_{\text{JB}}} - 1 \right] \times 100\%. \quad (33)$$

The corrections are small for small  $x_{\text{JB}}$  and  $y_{\text{JB}}$ . With rising  $y_{\text{JB}}$ , they may become large. In magnitude, they fall in between the leptonic and the considerably smaller hadronic corrections.

In the table, we also show numerical results from another calculation, which is restricted to the leading logarithmic approximation (LLA). These numbers have been taken from table 3 in [10] and are based on formulae published in [11]. The agreement is as good as one can expect for a LLA calculation. It is much better than has been observed for the case of leptonic variables, where certain kinematic singularities which are not related to the large logarithms from the particle masses, are much stronger pronounced. A naive conclusion from the comparison could be that experimentalists who apply radiative corrections to data should restrict to the use of the LLA formulae, thus optimizing the numerical computings. Let us remind here that this would not be too advantageous since both the exact  $\mathcal{O}(\alpha)$  and the approximated formulae are one-dimensional integrals.

In figure 1, we show the same corrections, but now including soft photon exponentiation (SPE; broken curves). Its inclusion effects the cross sections only there, where the corrections are numerically large, i.e. near  $y = 1$ .

*To summarize*, in this letter the complete  $\mathcal{O}(\alpha)$  QED corrections, including soft photon exponentiation, to deep inelastic  $ep$  scattering are obtained in JB variables for the first time in form of a semi-analytical expression. The numerical improvement compared to a LLA calculation, which also contains a one-dimensional integration, is minor.

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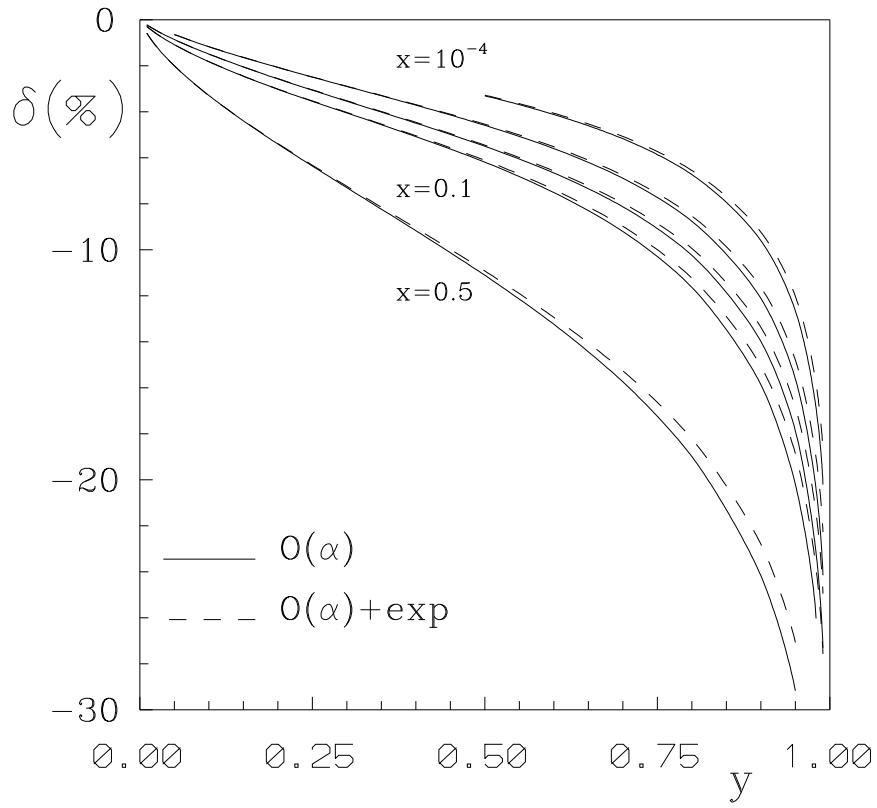


Figure 1: *Leptonic radiative corrections  $\delta$  of  $\mathcal{O}(\alpha)$  without (solid curves) and with (broken curves) soft photon exponentiation in JB variables; parameter:  $x_{\text{JB}} = 0.0001, 0.001, 0.01, 0.1, 0.5$ .*